

## Unit IV - Inference

### I. Inference for Categorical Data: Proportions

#### A. One Sample z-interval for a population proportion:

1. Verify the conditions for calculating confidence intervals for a population proportion:
  - a. To check that the sampling distribution of  $\hat{p}$  is approximately normal (shape):  
 **$n\hat{p} \geq 10$  and  $n(1 - \hat{p}) \geq 10$**  (number of successes and failures are at least 10)
  - b. To check that the sampling method is unbiased (so sampling distribution of  $\hat{p}$  will center at parameter):  
sample should be **SRS from population of interest** (discuss IN CONTEXT!!)
  - c. independence: If sampling without replacement,  $n < 10\%$  of population.

#### 2. Make Confidence Interval:

- a. On formula sheet:  $statistic \pm (critical\ value)(standard\ error\ of\ statistic)$

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

- b. Where  $\hat{p}$  is the midpoint of the interval, also called the 'point estimate'
  - c.  $z^*$  is the critical value, determined by the confidence level, the boundaries that have \_\_\_\_\_% **between** them, according to the normal distribution.
  - d.  $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$  is the standard error (estimate of the CLT standard deviation)
  - e.  $z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$  is the margin-of-error or half-width of the interval. A margin of error gives how much a value of a sample statistic is likely to vary from the value of the corresponding population parameter.
3. The formula for margin of error can be rearranged to solve for  $n$ , the minimum sample size needed to achieve a given margin of error. For this purpose, use a given estimate for  $\hat{p}$  or use  $\hat{p} = 0.5$  if none is given in order to find an upper bound for the sample size that will result in a given margin of error. (Round sample size up to whole number and include units on your answer.)
  4. Interpret a Confidence Interval:  
**I am \_\_\_\_\_% confident that the actual proportion of \_\_\_\_\_ (population) that \_\_\_\_\_ (variable) is between \_\_\_\_\_ and \_\_\_\_\_.**
  5. Interpret the confidence level (why are you \_\_\_\_\_% confident?):  
**If I repeatedly take samples of size \_\_\_\_\_ (n) from the population and create \_\_\_\_\_% confidence intervals for each. In the long run, approx. \_\_\_\_\_% of the intervals created would capture the actual proportion (p).**

Note: A confidence interval for a population proportion either contains the population proportion or it does not, because each interval is based on random sample data, which varies from sample to sample.

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6. Know what affects the width of an interval:
  - a. When all other things remain the same, the width of the confidence interval for a population proportion tends to decrease as the sample size increases. For a population proportion, the width of the interval is proportional to  $\frac{1}{\sqrt{n}}$
  - b. For a given sample, the width of the confidence interval for a population proportion increases as the confidence level increases.
  - c. The width of a confidence interval for a population proportion is exactly twice the margin of error (half-width).
  
- B. One sample z-test for population proportions:
  1. Hypotheses:
    - a. The null hypothesis is the situation that is **assumed to be correct** unless evidence suggests otherwise, and the alternative hypothesis is the situation for which evidence is being collected. The null hypothesis specifies a value for the population proportion, usually one indicating no difference or effect.
    - b. For hypotheses about parameters, the null hypothesis contains an equality reference ( $=$ ,  $\geq$ , or  $\leq$ ). Although the null hypothesis for a one-sided test may include an inequality symbol, it is still tested at the boundary of equality ( $=$ ). The alternative hypothesis contains a strict inequality ( $<$ ,  $>$ , or  $\neq$ ). The type of inequality in the alternative hypothesis is based on the question of interest. Alternative hypotheses with  $<$  or  $>$  are called one-sided, and alternative hypotheses with  $\neq$  are called two-sided.
    - c. The null hypothesis for a population proportion is  $H_0: p = p_o$ , where  $p_o$  is the null hypothesized value for the population proportion.  
A one-sided alternative hypothesis for a proportion is either  $H_a: p < p_o$  or  $H_a: p > p_o$ . A two-sided alternate hypothesis is  $H_a: p \neq p_o$ .
  
  2. Verify the conditions for performing a significance test for a population proportion:
    - a. To check that the sampling distribution of  $\hat{p}$  is approximately normal (shape): Assuming  $H_0$  is true,  **$np_o \geq 10$  and  $n(1 - p_o) \geq 10$**  (expected number of successes and failures are at least 10)
    - b. To check that the sampling method is unbiased (so sampling distribution of  $\hat{p}$  will center at parameter:  
sample should be **SRS from population of interest** (discuss IN CONTEXT!!)
    - c. independence: If sampling without replacement,  $n < 10\%$  of population.
  
  3. Calculate a test statistics and the p-value: (Draw a picture and think CLT!!)
    - a. On the formula sheet:  $test\ statistic = \frac{statistic - parameter}{standard\ deviation\ of\ the\ statistic}$
    - b. 
$$z = \frac{\hat{p} - p_o}{\sqrt{\frac{p_o(1-p_o)}{n}}}$$

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- c. find p-value using normal table or calculator.
- d. If asked to interpret a p-value:  
 The p-value is the proportion of values for the null distribution that are as extreme or more extreme than the observed value of the test statistic. This is:
  - i. The proportion at or above the observed value of the test statistic, if the alternative is  $>$ .
  - ii. The proportion at or below the observed value of the test statistic, if the alternative is  $<$ .
  - iii. The proportion less than or equal to the negative of the absolute value of the test statistic plus the proportion greater than or equal to the absolute value of the test statistic, if the alternative is  $\neq$ .
  - iv. **A p-value is the probability of obtaining a test statistic as or more extreme than the observed test statistic when the null hypothesis is assumed to be true.**  
 It's basically the probability that you randomly got your sample result from that population.

### 4. Conclusion:

- a. A formal decision explicitly compares the p-value to the significance level,  $\alpha$ . If the **p-value  $\leq \alpha$ , reject the null hypothesis**. If the **p-value  $> \alpha$ , fail to reject the null hypothesis**.
- b. Rejecting the null hypothesis means there is sufficient statistical evidence to support the alternative hypothesis. Failing to reject the null means there is insufficient statistical evidence to support the alternative hypothesis. The conclusion about the alternative hypothesis must be stated in context.
- c. A significance test can lead to rejecting or not rejecting the null hypothesis, but can never lead to concluding or proving that the null hypothesis is true. Lack of statistical evidence for the alternative hypothesis is not the same as evidence for the null hypothesis.
- d. Small p-values indicate that the observed value of the test statistic would be unusual if the null hypothesis and probability model were true, and so provide evidence for the alternative. The lower the p-value, the more convincing the statistical evidence for the alternative hypothesis.
- e. p-values that are not small indicate that the observed value of the test statistic would not be unusual if the null hypothesis and probability model were true, so do not provide convincing statistical evidence for the alternative hypothesis nor do they provide evidence that the null hypothesis is true.
- f. Writing Conclusion:

	<b>p-value <math>\leq \alpha</math></b>	<b>p-value <math>&gt; \alpha</math></b>
Significant?	Yes	No
Reject $H_0$ ?	Yes	No
Evidence to support $H_a$ ?	Yes	No

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With a p-value of \_\_\_\_\_, this is/isn't significant at the \_\_\_\_ level.

I reject/fail to reject  $H_0$ .

There is/isn't enough evidence to say  $H_a$  in context (answer the question in the problem).

### 5. Types of Errors:

Actual Population Value		
	$H_0$ is True	$H_0$ is false ( $H_a$ is true)
Reject $H_0$	Type I Error	Correct Decision
Fail to Reject $H_0$	Correct Decision	Type II Error

- a. A Type I error occurs when the null hypothesis is true and is rejected by the evidence (false positive).
- b. A Type II error occurs when the null hypothesis is false and is not rejected by the evidence (false negative).
- c. The significance level,  $\alpha$ , is the probability of making a Type I error, if the null hypothesis is true.
- d. The power of a test is the probability that a test will correctly reject a false null hypothesis.
- e. The probability of making a Type II error =  $1 - \text{power}$ .
- f. Whether a Type I or a Type II error is more consequential depends upon the situation.
- g. Since the significance level,  $\alpha$ , is the probability of a Type I error, the consequences of a Type I error influence decisions about a significance level.
- h. The power of a test increases when any of the following occurs, provided the others do not change: (\* because the probability of a Type II is  $1 - \text{power}$ , this probability automatically decreases when the power increases.)
  - i. Sample size(s) increases.
  - ii. Significance level ( $\alpha$ ) of a test increases.
  - iii. Standard error decreases. (most relevant to means)
  - iv. True parameter value is farther from the null.

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### II. Inference for Quantitative Data: Means

#### A. t-distribution:

1. When  $s$  is used instead of  $\sigma$  to calculate a test statistic, the corresponding distribution, known as the t-distribution, varies from the normal distribution in shape, in that more of the area is allocated to the tails of the density curve than in a normal distribution. (still mound shaped, but wider than a normal curve.)
2. Degrees of freedom (d.f.) =  $n - 1$
3. As the degrees of freedom increase, the area in the tails of a t-distribution decreases.
4. At  $df = \infty$ , the critical values for t are the same as for z.

#### B. one-sample t-interval for a mean

1. Verify the conditions for calculating confidence intervals for a population mean:
  - a. To check that the sampling distribution of  $\bar{x}$  is approximately normal (shape):  
 **$n \geq 30$  or the population is normal**
    1. Check if  $n \geq 30$
    2. If not, see if problem stated the population is known to be normal.
    3. Finally, the distribution of the sample data should be free from strong skewness and outliers. You must graph the data in order to discuss this.
  - b. To check that the sampling method is unbiased (so sampling distribution of  $\hat{p}$  will center at parameter:  
sample should be **SRS from population of interest** (discuss IN CONTEXT!!)
  - c. independence: If sampling without replacement,  $n < 10\%$  of population.
2. Make Confidence Interval:
  - a. On formula sheet:  $statistic \pm (critical\ value)(standard\ error\ of\ statistic)$   
$$\bar{x} \pm t^* \frac{s}{\sqrt{n}}$$
  - b. Where  $\bar{x}$  is the midpoint of the interval, also called the 'point estimate'
  - c.  $t^*$  is the critical value, determined by the confidence level, the boundaries that have \_\_\_\_\_% **between** them, determined by degrees of freedom on the t-table. If the d.f. are not on the table, use the smaller d.f.
  - d.  $\frac{s}{\sqrt{n}}$  is the standard error (estimate of the CLT standard deviation)
  - e.  $t^* \frac{s}{\sqrt{n}}$  is the margin-of-error or half-width of the interval.
3. Interpret a Confidence Interval:  
**I am \_\_\_\_\_% confident that the actual mean \_\_\_\_\_ (variable) \_\_\_\_\_ for \_\_\_\_\_ (population) \_\_\_\_\_ is between \_\_\_\_\_ and \_\_\_\_\_.**

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### C. One sample t-test for means:

1. The null hypothesis for a one-sample t-test for a population mean is  $H_0: \mu = \mu_0$ , where  $\mu_0$  is the hypothesized value. Depending upon the situation, the alternative hypothesis is  $H_a: \mu < \mu_0$ ,  $H_a: \mu > \mu_0$ , or  $H_a: \mu \neq \mu_0$ .

2. Verify conditions are met: same as for confidence interval.

3. test statistic is a t-score since we usually don't know  $\sigma$  and are substituting  $s$ .

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

4. the p-value is found using the t-table and the d.f. Look for the t-score on the d.f. row, and the p-values are at the top of the table. You will usually be reporting that the p-value is between two numbers.

\*\*Interpretations, conclusions, types of errors, what makes intervals wider/narrower, how power is affected, etc. are all the same for means as for proportions.